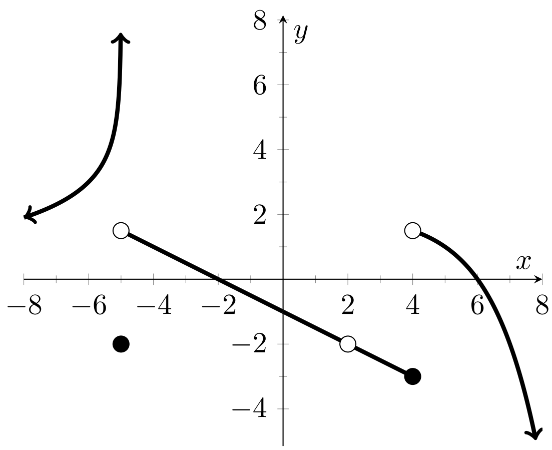
Math 251 Fall 2017 **Quiz #3, September 20**

Name: Solutions

There are 25 points possible on this quiz. This is a closed book quiz. Calculators and notes are not allowed. **Please show all of your work!** If you have any questions, please raise your hand.

Exercise 1. (5 pts.) Consider the function $f(x)$ with graph given below.



a.) List any values a where $\lim_{x\to a} f(x)$ fails to exist.

$-5, 4$

b.) List any values x where $f(x)$ fails to be continuous. Describe the type of discontinuity at each such value a.

-5 is an infinite discontinuity
2 is removable and
4 is a jump discontinuity.

Exercise 2. (4 pts.) Evaluate $\lim_{x\to 5} \dfrac{5x - x^2}{x^2 - 6x + 5}$.

$$\lim_{x\to 5} \frac{5x-x^2}{x^2-6x+5} = \lim_{x\to 5} \frac{x(5-x)}{(x-5)(x-1)} = \lim_{x\to 5} \frac{-x}{x-1}$$

$$= \frac{\lim_{x\to 5} -x}{\lim_{x\to 5} x-1} = \frac{-5}{4}$$

Exercise 3. (4 pts.) Evaluate $\lim_{x\to 1} \dfrac{\frac{1}{x^2} - 1}{x - 1}$.

$$\lim_{x\to 1} \frac{\frac{1}{x^2}-1}{x-1} = \lim_{x\to 1} \frac{\frac{1-x^2}{x^2}}{x-1} = \lim_{x\to 1} \frac{\frac{(1-x)(1+x)}{x^2}}{x-1}$$

$$= \lim_{x\to 1} \frac{-(1+x)}{x^2} = \frac{\lim_{x\to 1} (-x-1)}{\lim_{x\to 1} x^2} = \frac{-2}{1} = -2.$$

Exercise 4. (5 pts.) Consider the function

$$f(x) = \begin{cases} x+2 & x<2 \\ 1 & x=2 \\ \frac{16}{x^2} & x>2 \end{cases}$$

a.) Evaluate $\lim_{x\to 2} f(x)$.

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (x+2) = 4$ and

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} \frac{16}{x^2} = \frac{\lim_{x\to 2^+} 16}{\lim_{x\to 2^+} x^2} = \frac{16}{4} = 4.$

they agree, so $\lim_{x\to 2} f(x) = 4.$

b.) Explain why $f(x)$ fails to be continuous at $x=2$.

$\lim_{x\to 2} f(x) = 4 \neq 1 = f(2).$

Exercise 5. (4 pts.) Using complete sentences, explain why the function $f(x) = 2 + x^3 + \sin x$ has a zero on the interval $[-\pi, \pi]$.

Note that $f(-\pi) = 2 - \pi^3 + 0 < 0$ while $f(\pi) = 2 + \pi^3 + 0 > 0$. Also note that $f(x)$ is continuous on $[-\pi, \pi]$. thus, by the Intermediate Value theorem, there exists $-\pi < c < \pi$ such that $f(c) = 0$.

Exercise 6. (3 pts.) If $x^2 \le g(x) \le x^4 - x^2 + 1$ for all x, evaluate $\lim_{x\to 1} g(x)$. Justify your answer.

Observe that $\lim_{x\to 1} x^2 = 1$ and $\lim_{x\to 1}(x^4 - x^2 + 1) = 1-1+1 = 1$, so by the Squeeze theorem, $\lim_{x\to 1} g(x) = 1.$